The “Growing Method” of Eddy Identification and Tracking in Two and Three Dimensions

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1. Introduction

The method of eddy identification and tracking using sea surface height (SSH) described in Chelton *et al.* (2011) has been modified in a manner inspired by Williams *et al.* (2011). The original method summarized in detail in Appendix B.2 of Chelton *et al.* (2011) was based upon finding closed contours of SSH encircling a set of grid points, or *pixels*, which, subject to a set of five conditions, defined an eddy. The use of a contouring algorithm to define these regions was cumbersome, and is not easily extensible to three dimensions for identifying eddies in ocean model simulations. The simpler method of Williams *et al.* (2011), modified as described here, finds anticyclonic eddies by starting with the pixel at a local maximum of SSH and successively finding all neighboring pixels whose SSH values lie above a sequence of successively decreasing thresholds. This “growth” of the eddy interior is continued until any one of the five criteria for compact and coherent structure described below is violated. Cyclonic eddies are defined by using the negative of the SSH field and proceeding likewise. The five criteria used were chosen to yield eddies that are statistically similar to those using the old method of Chelton *et al.* (2011).

This “growing method” of eddy identification is not restricted to SSH. It can be applied to other fields used to characterize eddies, e.g., the Okubo-Weiss parameter considered by Williams *et al.* (2011) and many others, including our early eddy analysis (Chelton et al., 2007). As discussed in Appendix B.5 of Chelton *et al.* (2011), however, use of the Okubo-Weiss parameter is inferior to use of SSH for eddy identification from altimetric data because of the amplification of noise by the double differentiation required to compute the Okubo-Weiss parameter from SSH.

A distinct advantage of the growing method of eddy identification is that it can be directly applied to 3-dimensional fields obtained from ocean models, and can thus be used to explore the vertical structure of eddies. The 3-dimensional analog of SSH used here for 2-dimensional eddy identification is subsurface pressure anomalies.

2. Eddy Identification in Two Dimensions from Sea-surface Height

We consider an eddy to be a propagating, compact, coherent structure in the space-time SSH fields. The SSH signature of the eddy at a specific time is referred to as a realization of the eddy. In this section, we present our definition of an eddy realization and describe the method for identifying all of the eddy realizations in a gridded field of spatially high-pass filtered SSH at a given time step. The purpose of the spatial high-pass pre-filtering is to help the eddy identification procedure by removing large-scale signals in SSH that are unrelated to the mesoscale eddies that are of interest. The filtering we apply has half-power filter cutoff wavelengths of 20° of longitude by 10° of latitude. These spatially high-pass filtered SSH fields are referred to simply as SSH hereafter, with the filtering implicit.

The eddy identification procedure summarized in this section is applied independently to the filtered SSH field at each time step in the data record. The procedure for tracking the eddies from one time step to the next is summarized in Section 3.

2.1 *Definitions*

At each particular time of the data record, we analyze a discretized, 2-dimensional field of SSH, $h(i, j)$, where each index i corresponds to a specific longitude $x(i)$, and each index j corresponds to a specific latitude $y(j)$, i.e., a grid of pixels, each with its specified SSH value. In two dimensions, the *neighbors* of a given pixel with indices (i, j) are generally taken here to be the the four neighbors with indices $(i-1, j)$, $(i+1, j)$, $(i, j-1)$, and $(i, j+1)$. In three dimensions, six neighbors are used.

For identification of anticyclonic eddies (concave downward SSH), a pixel (i_{ext}, j_{ext}) is defined to be a local positive extremum if $h(i_{ext}, j_{ext})$ is greater than or equal to the SSH values of its four neighbors. Note that this definition allows for the possibility of a positive extremum with negative SSH value that can occur for an anticyclone located in a region of residual large-scale negative SSH from processes distinct from mesoscale eddies.

For identification of cyclonic eddies (concave upward SSH), a pixel (i_{ext}, j_{ext}) is defined analogously to be a local negative extremum if $h(i_{ext}, j_{ext})$ is less than or equal to the SSH values of its four neighbors.

For a given connected set of pixels (defined below), a single pixel is *interior* to a mesoscale eddy if all of its neighbors are also in the set. Otherwise, it is referred to here as an *edge* pixel.

Consider an anticyclonic eddy with a local maximum SSH at grid location (i_{ext}, j_{ext}) and a specified threshold SSH value $h_t \leq h(i_{ext}, j_{ext})$. Define $E(i_{ext}, j_{ext}, h_t)$ as the connected set of pixels $(i_l, j_l),\ l = 1, \ldots, n$ that contains (i_{ext}, j_{ext}) and satisfies $h(i_l, j_l) \geq h_t,\ l = 1, \ldots, n$. The procedure described below for anticylonic eddies is applied to identify cyclonic eddies by considering $-h(i, j)$ rather than $h(i, j)$. We seek h_b, the minimum value of incrementally decreasing thresholds h_t such that $E(i_{ext}, j_{ext}, h_b)$ satisfies the following criteria:

1) $n \leq n_{max}$, a specified maximum number of pixels in $E(i_{ext}, j_{ext}, h_t)$.
2) $n \geq 2$, a minimum of two interior pixels in $E(i_{ext}, j_{ext}, h_t)$.
3) No pixel in $E(i_{ext}, j_{ext}, h_t)$ can have as a neighbor a pixel that belongs to another eddy.
4) $E(i_{ext}, j_{ext}, h_t)$ is simply connected, i.e., there are no “holes” in the eddy.
5) Let $d(i_k, j_k, i_l, j_l)$ be the distance between pixels (i_k, j_k) and (i_l, j_l). Then the maximum value of $d(i_k, j_k, i_l, j_l)$ over all pairs of edge pixels in $E(i_{ext}, j_{ext}, h_t)$ must be less than a specified value d_{max}.

The compact and coherent structure $E(i_{ext}, j_{ext}, h_b)$ is defined to be an eddy realization with *basal* SSH value of h_b. The set of edge pixels in $E(i_{ext}, j_{ext}, h_b)$ defines the outer perimeter of the eddy realization.

2.2 *Details of the Pixel-Growing Algorithm*

2.2.1 Growing the Eddy Interior

In practice, h_b is determined by generating ("growing") a sequence of pixel sets at different thresholds separated by a threshold increment δ: $E_l = E(i_{ext}, j_{ext}, h_l)$, where $h_l = h_0 - l\delta$, $h_0 = l_0\delta$, l_0 is the largest integer such that $h_0 \leq h(i_{ext}, j_{ext})$ and $l = 0, 1, 2, \ldots$. Given a set of pixels E_l, the next set E_{l+1} is computed by finding all of the neighbors of the edge pixels in E_l that exceed h_{l+1}, which are then added to E_l. This process is repeated using newly added pixels until no new pixels are found to be connected to E_l that exceed the incrementally decreassing threshold h_{l+1}.

2.2.2 Initializing the Procedure

Eddy realizations are identified by growing sets of pixels from the single pixels at the local maxima in $h(i,j)$ and $-h(i,j)$ for anticyclonic and cyclonic eddies, respectively. These maxima are sorted into decreasing size and eddy realizations are obtained from successively smaller inital values of h or $-h$ without regard to polarity.

2.2.3 Finding h_b: Stopping the Pixel-Growing Procedure

At each step in the sequence E_l, each of the five criteria listed in Section 2.1 is checked. When at least one of these criteria is violated, the sequence is stopped at the previous step, E_{l-1}. The fourth criterion is implemented by searching for any non-eddy pixel that has at least two of the edge pixels of E_l arrayed to the north and south along the same longitude (the i index) and at least two more arrayed to the west and east along the same latitude (the j index). When this occurs, the eddy is either not simply connected, or there is an "embayment" along the eddy perimeter, neither of which we consider to be an acceptable compact and coherent eddy structure. For the first few thresholding steps, the fourth criterion may halt eddy growth prematurely. Thus, rather than stopping eddy growth, a flag denoting connectivity is carried along until eddy growth is stopped by one or more of the other four criteria, at which point the set of connected pixels that define the eddy is contracted back to the previous set in the sequence where simple connectivity obtains.

2.3 *Characterizing the Eddy: Amplitude, Scale and Rotational Speed*

Given a set of connected pixels $E(i_{ext}, j_{ext}, h_b)$ in a map of SSH that define an eddy realization as outlined in Section 2.2, there are a number of parameters of interest:

1) The longitude and latitude coordinates (x_c, y_c) of the SSH-based eddy centroid, which are defined to be

$$x_c \equiv \frac{\sum_{(i,j)\in E}\ x(i)h(i,j)}{\sum_{(i,j)\in E}\ h(i,j)}$$

$$y_c \equiv \frac{\sum_{(i,j)\in E}\ y(j)h(i,j)}{\sum_{(i,j)\in E}\ h(i,j)}\ .$$

   For this definition to make sense, $h(i,j)$ must be positive for all $(i,j) \in E$. If this is not the case, the SSH within the eddy must be shifted by a constant.
2) The amplitude A, which is defined to be the difference between the extremum SSH value of $h(i_{ext}, j_{ext})$ and the average of SSH over the edge pixels that define the outer perimeter of the eddy.
3) The effective radius scale L_{eff}, which is defined to be the radius of a circle with area equal to that of the set of connected pixels $E(i_{ext}, j_{ext}, h_b)$.
4) At each threshold $h_l \geq h_b$, the average of geostrophic speed over the edge pixels of E_l is found. The rotational or axial speed U of the eddy is defined to be the maximum such average. The threshold SSH at which this maximum average occurs is defined to be h_U. The subset of connected pixels $E(i_{ext}, j_{ext}, h_U)$ is referred to as the *speed core* of the eddy.
5) The speed-based radius scale L, which is defined to be the radius of a circle with area equal to that of the set of connected pixels of the speed core $E(i_{ext}, j_{ext}, h_U)$.

2.4 *Application to the AVISO SSH Fields*

The algorithm defined above was applied to the AVSIO DT-2014 daily SSH fields on a 1/4° × 1/4° grid using a threshold increment of $\delta = 0.25$ cm and a maximum number of connected pixels n_{max} =2000. The maximum distance bound in the fifth criterion in Section 2.1 was set to $d_{max} = 400$ km for latitudes greater than ±25°. Within the tropical latitude band ±25°, the maximum distance bound increased equatorward cosinusoidally to a maximum of $d_{max} = 700$ km at the equator. An additional constraint that eddies must have amplitude $A \geq 1$ cm was imposed.

3. Tracking Eddies

3.1 *The Tracking Procedure*

A propagating eddy is formed by pairing eddy realizations from one time step to the next time step. To define how this is done, modify the notation above so that $E(i,t)$ is the i^{th} eddy realization found at time step t, with corresponding centroid $[x_c(i,t), y_c(i,t)]$, amplitude $A(i,t)$ and effective radius $L_{eff}(i,t)$. Two eddy realizations $E(i,t)$ and $E(j,t+1)$ are paired if:

1. Within a defined spatial search region S, the j^{th} eddy realization at time step $t+1$ with centroid location $[x_c(j,t+1), y_c(j,t+1)]$ is the closest to the centroid $[x_c(i,t), y_c(i,t)]$ at time step t.
2. The amplitude $A(j,t+1)$ and effective radius scale $L_{eff}(j,t+1)$ fall within the ranges $\alpha^{-1}A(j,t+1) \leq A(i,t) \leq \alpha A(j,t+1)$ and $\alpha^{-1}L_{eff}(j,t+1) \leq L_{eff}(i,t) \leq \alpha L_{eff}(j,t+1)$ for a specified constant $\alpha \geq 1$.

In step 1, the proximity search for paired realizations is restricted to the search region S in order to reduce the likelihood of jumping from one eddy track to another. The ranges of allowable amplitude and radius scale variation from one time step to the next that is controlled by the parameter α in step 2 accommodate natural variability and noise in the SSH fields. A value of $\alpha = 2.5$ was used for our eddy dataset.

Tracking begins at the first time step and proceeds by pairing eddy realizations at subsequent time steps. At any time step, a new eddy may be initiated at a realization that is unpaired with a realization at the previous time step. An eddy is terminated when a realization is not paired at the next time step. Typically, only eddies longer than a specified minimum lifetime are retained. A minimum lifetime of 4 weeks was used for our eddy dataset.

Tracking in three dimensions is the same as in two dimensions, using the centroids, effective radii and amplitudes obtaining at the depth $z(k_{ext})$ of the local maximum in the field.

3.2 *Application to the AVISO SSH Fields*

The DT-2014 AVISO SSH fields have been analyzed using the tracking method summarized in Section 3.1 applied to the eddy realizations identified as summarized in Section 2. The fields were analyzed at the daily time intervals of the DT-2014 dataset using the parameters given in this section. The search region S is defined as the interior of an ellipse with zonally oriented major axis. The eastern extremum of the ellipse is 150 km to the east of the centroid location $[x_c(i,t), y_c(i,t)]$ of the current eddy, and the north-south semi-minor axis of the ellipse is 150 km. The western extremum of the ellipse is a distance e from the current eddy center, where e is never less than 150 km. In concert with the observed increase of propagation speeds with decreasing latitude, e is set to 1.75 times the distance that a long baroclinic Rossby wave can propagate in one week computed from the long Rossby wave phase speed based on the climatology of the Rossby radius of deformation in Chelton *et al.* (1998). The range of allowed amplitudes and radius scales for paired realizations is specified by setting $\alpha = 2.5$, which was chosen based on trial and error. In consideration of the ~35-day decorrelation time scale in the AVISO OI procedure, only eddies with lifetimes of at least 28 days were retained.

References

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