

# The Cycling Representer Method Applied to the Navy Coastal Model

Scott R. Smith, Hans E. Ngodock, and Gregg A. Jacobs

Naval Research Laboratory  
Stennis Space Center, MS  
smithsc@nrlssc.navy.mil



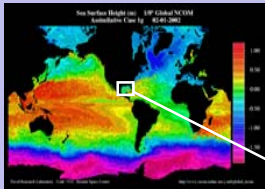
## (1) Motivation:

The cycling representer method is a technique for solving a highly nonlinear 4D-variational data assimilation problem. 4D-VAR combines observations with an ocean model in order to obtain an optimal solution that minimizes a cost function (weighted squared errors of data, initial conditions, boundary conditions, forcing, and model dynamics). However, in order to converge to the global minimum of this cost function, the ocean model (and its adjoint) must be linear. Ocean models, especially those that are designed to resolve baroclinic and mesoscale processes, are typically highly-nonlinear and must be linearized. Tangent linearization is a linearization method that is performed by expanding the nonlinear dynamics about a background field using the first order approximation of Taylor's expansion. The accuracy and stability of this tangent linearized model (TLM) can be a function of background accuracy, the level of nonlinearity of the model, complexity of the bathymetry, and the complexity of the flow field. Therefore, in small coastal domains the TLM is only going to be stable for relatively short periods of time.

In order to ensure TLM stability, and hence proper data assimilation, the cycling representer method is employed to split the time period of the assimilation problem into smaller cycles. The cycle length needs to be short enough for the TLM to be stable, but long enough to minimize the loss of information due to reducing the temporal correlation of the dynamic error. For each new cycle, a background is first created as a nonlinear forecast from the previous cycle assimilated solution. Then, data that falls within the time period of the new cycle is used to calculate a new assimilation solution. The cycling representer method has been demonstrated to drastically improve assimilation accuracy with both the Lorenz Attractor (Ngodock & Smith, 2007) and a reduced gravity ocean model (Ngodock & Smith, submitted). Now, this assimilation method is being applied to the Navy Coastal Ocean Model (NCOM). This assimilation system is demonstrated in the Mississippi Bight with an array of 14 ADCP moorings deployed from May 2004 to May 2005. The initial background field, around which the TLM is expanded, comes from an operational regional version of NCOM.

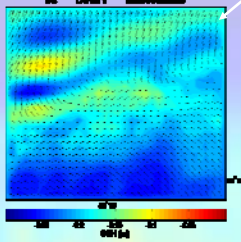
## (2) The Forward Ocean Model:

### Global Navy Coastal Ocean Model (NCOM)



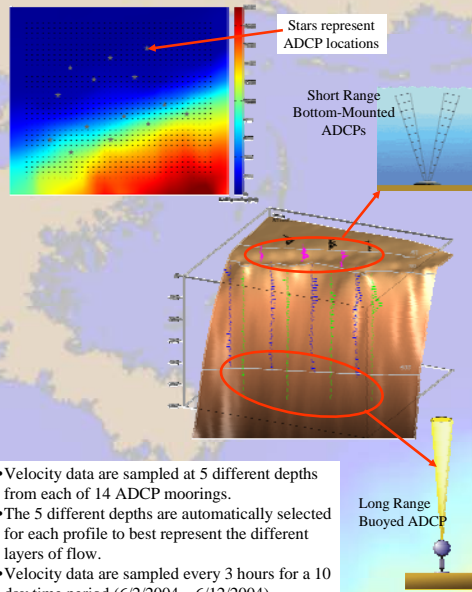
- Operational
- 1/8° resolution
- 40 sigma-Z layers
- leapfrog time stepping
- implicit treating of free surface
- 5 day forecast

### Nested NCOM for Mississippi Bight



- Ran nested model for 10 days (6/2/2004 - 6/12/2004)
- 30 X 34 grid points
- 40 sigma-Z layers
- 2.5 km resolution
- Solutions are used for the initial conditions, boundary conditions, and background for the 1<sup>st</sup> cycle of the cycling representer method

## (2) The Data:

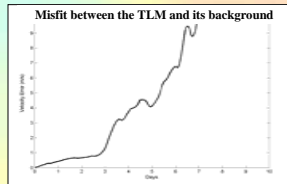


- Velocity data are sampled at 5 different depths from each of 14 ADCP moorings.
- The 5 different depths are automatically selected for each profile to best represent the different layers of flow.
- Velocity data are sampled every 3 hours for a 10 day time period (6/2/2004 - 6/12/2004).
- A total of 11,200 velocity measurements are assimilated.
- Measurements are represented through the measurement functionals as Gaussians.

## (3) Tangent Linearized Model (TLM):

NCOM dynamics are linearized about the background using the first order approximation of Taylor's expansion:  $A_{TL} \bar{x} = A \bar{x}_{BG} + \frac{d(A \bar{x}_{BG})}{d\bar{x}_{BG}} (\bar{x} - \bar{x}_{BG})$

Due to the high-nonlinearity of NCOM and the complexity of the flow field and bathymetry, the TLM is not stable and accurate for the entire 10 day time period.



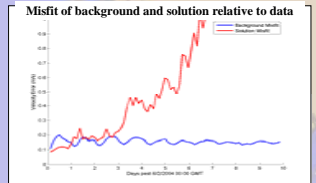
## (4) Assimilation Results:

The 4D-VAR assimilation problem is solved using the representer method with representer coefficients computed iteratively using a conjugate gradient. Covariances are correlated spatially with Gaussians and temporally with moving averages.

### Assimilation over the entire 10 day time period

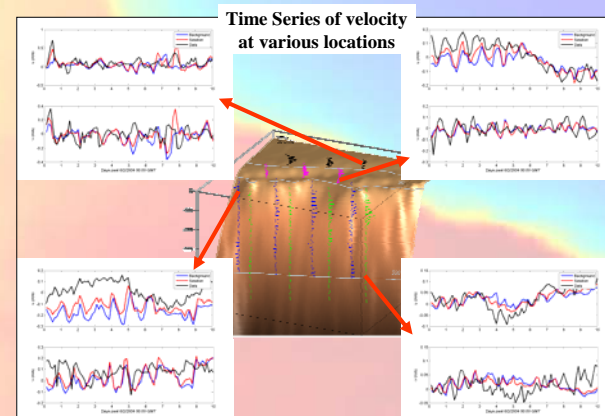
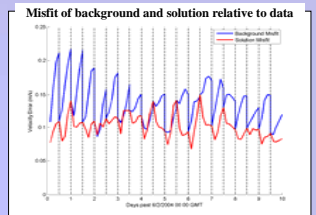
This result reveals that there is a direct correlation between the accuracy of the assimilated solution and the TLM stability. Since the TLM is only stable for 1-2 days, the assimilation problem must be shorter than this in order to ensure a stable and accurate solution.

It should be noted that this assimilation experiment did not converge. It was stopped after 104 iterations of the conjugate gradient, and a final sweep was performed with the best estimate of representer coefficients.



### Cycling the representer method with 12 hour cycles

By splitting up the problem into 12 hour cycles, the assimilated solution is stable and improving the forecast over the entire 10 day time period. For each cycle, a nonlinear forecast is performed with initial conditions taken from the previous solution and boundary conditions taken from the previous forecast. To accomplish this transition, each cycle assimilation runs an extra 3 hours into the next cycle. Then a time weighted average is used on the solution and previous forecast for these 3 hours to create smooth boundary conditions for the next forecast.



**Conclusion:** Even though the TLM of NCOM is only stable for 1-2 days, the cycling representer method can be used in conjunction with the conjugate gradient algorithm to achieve an improved analysis for an indefinite period of time. It is apparent that as the system cycles, the forecast and analysis errors are getting smaller. If the system were to continue to cycle beyond the 10 days shown here, these errors should converge to a minimum. It is our hypothesis that the reason this cycling representer experiment is taking a significant number of cycles to converge, is because of poor specification of the covariances, and the transition used to create the boundary conditions for each cycles forecast.