Relative Entropy and Predictability of Oregon Coastal Ocean Flows

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Introduction

The long term goals of the study are to address issues of uncertainty and predictability in Oregon coastal ocean flows. The uncertainty and predictability of the ocean circulation can be quantified by the relative entropy, which gives a measure of predictive information content based on ensemble distributions. The concept of the relative entropy is here explored in primitive-equation simulations of ocean circulation along the Oregon coast. The ensemble members are obtained by perturbing the initial temperature field with white noise, and then advancing the simulations for 60 days with periodic wind forcing that has a 5-day period and a southward mean.

Relative Entropy *R(p|q)*

The relative entropy $R(p\,|\,q)$ is defined as $R(p\,|\,q)=\int p(\mathrm{X})\cdot\log\frac{p(\mathrm{X})}{q(\mathrm{X})}d\mathrm{X}$ which a measure of the lack of information in one PDF (*q*) relative to other PDF (*p*), or the information in *p* relative to *q*. That is, It quantifies the amount of information that *p* provides beyond *q*.

When both *p* and *q* are assumed to be Gaussian, the relative entropy is decomposed into two components, *Signal* and *Dispersion*.

$$\text{Signal}=\frac{1}{2}\cdot\frac{(\mu_p-\mu_q)^2}{\sigma_q},\qquad \text{Dispersion}=\frac{1}{2}\cdot[\log\frac{\sigma_q}{\sigma_p}+\frac{\sigma_p}{\sigma_q}-1]$$

The signal is governed by the amplitude of the predicted mean field , that measures the contribution of the predicted signal size. Since the variance of climatology *q* is invariant, the dispersion is a measurement of the ensemble spread. A larger relative entropy implies more useful information supplied by the prediction *p*.

Model Simulation

- Regional Ocean Model Systems (ROMS)
- 229km offshore, 365km alongshore, 31 sigma-levels in vertical.
- Periodic channel on the South and North, Wall-boundary on the West
- 60-day simulations forced by 5-day periodic wind stress with southward mean
- 25 ensemble members perturbed temperatures in a initial state with white noise of amplitude 0.05
- Figures show temperatures at depth 5m, the non-perturbed initial state and a perturbed ensemble members, and wind stress forcing.

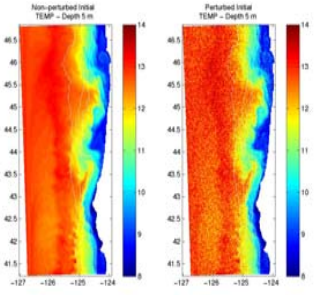

Figure 1 : Initial state and its perturbed state

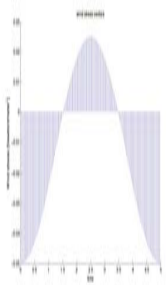

Figure 2 : Wind stress

Reference Distribution *q*

The means and variances for the reference distribution is obtained by averaging 25 ensemble members over 60 days. Left figure shows mean distribution at depth 5m and its standard deviation for Right figure

$$\mu_q=\frac{1}{N\cdot T}\sum_{t=1}^{T}\sum_{n=1}^{N}\mathrm{X}_t^n$$

$$\sigma_q=\frac{1}{N\cdot T-1}\sum_{t=1}^{T}\sum_{n=1}^{N}[\mathrm{X}_t^n-\mu_q]^2$$

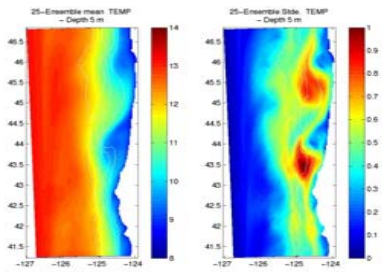

Figure 3 : Climatology mean and standard deviation

Ensemble Distribution *p*

The first two moments for the prediction distribution is obtained from the ensemble distribution. The ensemble spread grows wider with time.

$$\mu_p=\frac{1}{N}\sum_{n=1}^{N}\mathrm{X}_t^n,\qquad \sigma_p=\frac{1}{N-1}\sum_{n=1}^{N}[\mathrm{X}_t^n-\mu_p]^2$$

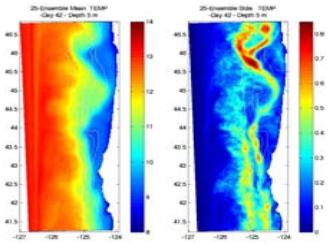

Figure 4 : Mean and standard deviation of ensemble distribution at Day 42

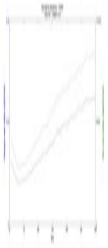

Figure 5 : Domain-averaged standard deviation over time

Relative Entropy

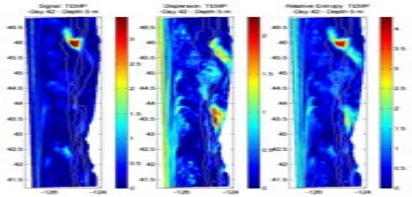

Figure 6 : A snap-shot of the relative entropy at day 42.

The relative entropy quantifies the amount of information in each ensemble distribution. So, a large value of *R* indicates more informative and accurate prediction whereas a small value of *R* shows poor prediction or same information from climatology *q*.

Color bar indicates magnitudes of the information gain beyond the *q*.

The magnitude of the relative entropy is mainly dominated by dispersion.

The signal contributes relatively more after day 40.

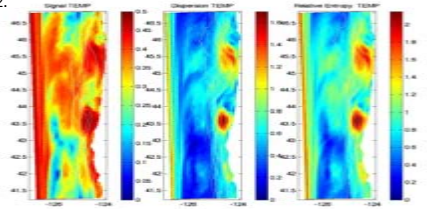

Figure 7 : Accumulated relative entropy for 60 days

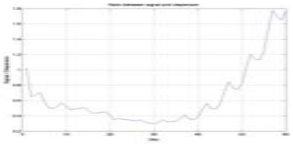

Figure 8 : Ratio between Signal and Dispersion

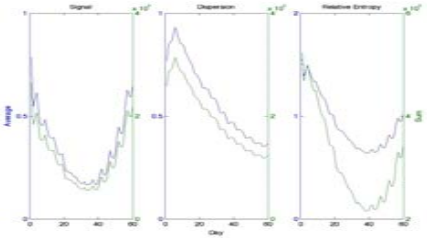

Figure 9 : Domain averaged signal, dispersion and relative entropy over time

Summary

- Relative entropy can be used to quantify predictive information content in ensembles of coastal ocean simulations
- Spatial variations in the ensemble variance and the relative entropy are related to topographic features
- For a climatology constructed from the time and ensemble mean and variance, the dispersion dominates the relative entropy during day 0~40, and the signal dominates after day 40

Future Work

- Explore dependence on the reference distribution *q* including normality assumption
- Relate ensemble statistics to dynamical processes including instabilities and topographic interactions.

Acknowledgement

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References

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