Comparing Glider Observed Velocities and Geostrophic Currents

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Abstract

Autonomous underwater vehicle gliders are a new cost effective method of studying oceanic processes. The Oregon coast's dominant coastal regime is the upwelling which occurs during the summer season via Ekman transport during which the net vertical volume of water is transferred 90° to the right which forces warm, surface waters offshore and brings cold, deep water to fill it's place. The upwelling regime creates a unique coastal environment in which the glider's are used to study it's effects. The glider travels at a relatively constant speed (26 cm/s) and measures observed water velocities using it's internal dead reckoning system. This project endeavors to compare calculated geostrophic velocities to the water velocities measured by the glider. Geostrophic currents rely solely on the balance between the Coriolis force (caused by the rotating Earth) and the pressure gradients present in the ocean. Therefore by evaluating the presence of geostrophic currents in observed velocities there would a better understanding of the geostrophic currents effects on the oceanic processes along Oregon's coast.

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1. Objective

To further characterize absolute and geostrophic currents along the Newport Hydrographic line using data collected from autonomous underwater gliders.

2. Introduction

The coastal upwelling regime, which regulates most of Oregon’s climate, is due to two variables: 1) Ekman transport of the surface water layer and 2) the effects of coastal boundaries. Upwelling is the transport process that enables the ocean to affect the coastal climate, biological diversity and coastal characteristics such as density and temperature. Upwelling occurs when a wind field provides enough stress/friction on the surface (Ekman) water layer which leads an Ekman spiral formation which transfers momentum down through the water layers with each layer traveling slower than the previous due to friction until motion stops (Ocean Circulation 1989). The wind pattern (north to south) along coastal Oregon during the summer season is particularly favorable to these conditions therefore causing the surface layers of water to move westerly from the shoreline therefore removing the warm coastal waters and revealing colder deeper water. Another important process is when a convergence of the surface (Ekman) water causes sinking of the surface layer, which is called “downwelling” (Ocean Circulation 1989). The understanding of how these two oceanic processes work can lead to valuable insight into the physical, chemical and biological structure of a volume of ocean.

The central Oregon coast has been well studied over the latter half of the 20th century using the Newport Hydrographic (NH) transect running along 44.65°N beginning with the TENOC (The Next Ten Years of Oceanography) program beginning in 1961 and...
ending in 1971. It was sporadically used as a data/sample collecting grounds until 1997 when the LTOP (Northeast Pacific Long Term Observations Program) began an intensive study of the NH transect that concluded in 2003. Data such as temperature, salinity vs. density as well as chemical and biological samples were taken during the LTOP program (Huyer et. al 1983).

Comparative analysis of the data collected by the two programs reveals that the seasonal cycling of surface layer properties is very strong with exceptional differences when comparing summer data. The summer regime data from LTOP demonstrated far warmer and fresher water in the surface layer (0-100m) than in TENOC with higher steric heights over the continental margin as well (Huyer et. al 2006). Whereas winter data showed little variance between the two studies, it can be concluded however that the Subarctic invasion in 2002 and El Niño in 1997-98 greatly influenced the differences in climate seen today (Huyer et. al 2006). LTOP and TENOC have provided valuable information about the seasonal cycling of upwelling regimes and agree that coastal currents reflect alongshore advection (Huyer 1983, Huyer et. al 2006).

The upwelling occurring off the Oregon coast is a seasonal event that is governed by alongshore velocities due to a poleward undercurrent at the bottom of the water column and a surface, southward “coastal jet” (Huyer et. al 1983, Huyer 2006). The studies of coastal regimes off Oregon have been isolated to the small scale off the Oregon coast as well as some of northern California, it is yet to be seen if there is a similar upwelling regime along the coast south of Oregon (Huyer 1983).

Currently a new method of characterizing the NH transect is being pursued using autonomous underwater gliders as instruments to gather oceanographic data. The gliders
are sent into the field for approximately three weeks and during that time follow the route of the NH transect, taking such sophisticated data as turbidity, oxygen content in the water, temperature, salinity as well as chlorophyll concentrations due to phytoplankton.

A glider operates using battery power in order to manipulate the ballast of the vessel, which allows it to sink and rise while the wings attached to the glider help to transfer the vertical motion into horizontal motion. It is also equipped with GPS and an antenna to allow for data uploads to a main computer every six hours. This new instrument has the potential to allow more long-term observational studies to take place and therefore provide new information regarding the physical ocean processes.

3. Experimental Description

The majority of large scale ocean currents outside of the surface Ekman layer are produced by a balance between a horizontal pressure gradient force and the Coriolis force (Equation 1) which is caused by the rotation of the earth. This is known as the geostrophic balance. A geostrophic current is unique because it’s driven by a pressure gradient which results in a perpendicular flow of water at a 90° angle to the pressure gradient. If there is a pressure gradient from east to west (from high to low) there is a resulting northward geostrophic current. This due to the influence of the rotation of the earth, without rotation the fluid would simply flow from high to low pressure.

A comparison between the observed glider velocities and calculated geostrophic currents demonstrates the variability between the water velocities measured by the glider and the overall geostrophic flow calculated from the geostrophic and hydrostatic balances (Equations 2, 3 and 4). The glider velocities can be checked against the graphed and
averaged geostrophic velocities to discern any discrepancies in the velocity measurements of the glider as well as determine how much the movement of water is affected by other processes besides the geostrophic flow. Discrepancies may arise from variations in Ekman transport in the upper layers of the water or from changing atmospheric conditions (wind).

**Glider Operation**

The Slocum electric glider was used in this study from Webb Research based in Massachusetts. The unique property of these coastal gliders is it's energy efficient due to it's ability to manipulate it's buoyancy by intaking water through it's nose which can again be expelled therefore changing the mass of the glider. This causes the glider to sink and rise in the water. The pitch is also controlled by using the moveable battery packs within the nose of the glider which move towards the stern of the glider or towards the nose in it's front compartment. This allows the glider to change it's glide angle while the glider's wings serve to change it's vertical motion into horizontal motion (Webb 1997).

The glider dives for approximately six hour periods for an entire three week mission. It surfaces after every six hours in order to relay information to the Iridium satellite as well as receive new instructions by downloading the information using the antenna which sits on top of it's tail. There are three sensors attached to the glider which collect data: 1) the CTD (conductivity, temperature and depth) sensor, 2) optics and 3) DO (dissolved oxygen sensor) (Webb 1997).

**Observed Glider Velocity**

Water velocity measurements calculated by the glider are calculated using the GPS position data from the glider which are then compared against the modeled
geostrophic velocities. Velocities of the surface layer of water (upper Ekman layer) were averaged using data from individual dive periods of the glider (*Equation 2*) which normally lasted approximately six hours.¹ From the various data collected, the speed and geographic position of the glider were used to compute the velocity of water through which it traveled. As seen in Figure 1, only one dive period was looked at (lasting approximately six hours) in contrast to an entire glider flight.

![Glider travel from August 8, 2006](image)

**Figure 1:** Glider travel from August 8, 2006 where red and light blue sections of the longitude (blue) and latitude (green) travel lines represents the period of glider travel discussed here.

The velocity of the upper Ekman layer, or surface layer, is estimated using GPS tracking and speed data each time the glider surfaces. This process is illustrated in

¹ File of processed DBD data used in analysis: 08-02-06\NH_2006080221917-deployment_bob.mat
Figure 2, where the glider collects it's starting coordinates at point one (P1/t1), upon submerging knows it's speed, heading (internal compass) and forward velocity. It dead reckons it's next point of emersion to be point 2 (P2/t2) however the GPS (through the Iridium connection) confirms a different location than what it had previously thought therefore creating a third point (P3/t2).

Figure 2: The calculation of the water's velocity can be derived from the glider's calculated velocity via start position (P1/t1), it's calculated position upon surfacing (P2/t2) and it's actual position (via GPS, P3/t2). The difference between the actual and calculated position (dotted line) is the velocity of the water. (Note: the scaling is arbitrary).

The third point (P3/t2) is the actual position of the glider after it has drifted according to currents in the water because the glider travels at an approximately constant speed during the entire time it's submerged. The distance(m) between these two points (P2 and P3) equals the integral of the velocity for a period of time therefore making possible the
calculation of the velocity of the water. It is the difference in position between point 2 and point 3 over the time submerged (one 6 hour dive period) that the water velocity is calculated by the glider.

The calculated distance between data points (longitudes and latitudes) was used in order to model the geostrophic currents present over the same dive periods as seen in Figure 1. This enables the distance traveled to be known between points of emersion and therefore analyze how the strength of currents present in the water during a dive period. Figure 3 displays both east/west (Vx) and north/south vertical (Vy) velocities calculated for the entire glider flight examined in this paper.

Figure 3: Velocity data from August 8, 2006 for the entire glider flight from which velocity indices 20202:25277 were identified as one six hour travel period of the glider which was used for the distance and geostrophic current calculations. The two boxed data points indicate the velocity period chosen for the comparison the modeled geostrophic currents.
The distance was calculated using the corresponding latitudes and longitudes which corresponded to the velocity indices of 20202 through 25277 for one dive period of the glider during August 8, 2006. The corresponding depth(m) values that the glider traveled within these indices were plotted against distance(km) values and shaded for the variables of potential density (PDen) and temperature(°C) in order to better characterize the volume of water being studied. As seen in Figures 4 and 5, The variations of potential density(kg/m^3) and temperature(°C) are examined over depth(m) and distance(km) and show the general trends of water layers being denser the deeper the depth (Figure 4) and less near the surface whereas the water layers are approximately 12°C near the surface and less than 8°C near the bottom of the glider's dive.

Figure 4: The potential density of the volume of water considered the trajectory traveled over the implied indices 20202:25277 for one six hour dive. The plot of the glider depth(m) over the distance(km) traveled illustrates the saw-tooth trajectory of the glider.
Figure 5: The temperature(°C) variability over the isolated glider dive is shown here again illustrated with the saw-tooth travel pattern of the glider by depth(m) versus distance(km) traveled by the glider. The temperature ranges from 8°C to 12 as displayed in the colorbar.

The experimental glider velocity data was taken during the regular dive sessions of the glider therefore produces a singular value for each dive. The overall velocity was averaged using an averaging operator (Equation 1) for velocities, \( v_1 \) when immersing and a \( v_2 \) when emerging from the water was therefore the calculations for the modeled geostrophic velocities also had to be written to represent average velocities.

\[
V = \left( \frac{1}{\Delta x} \right) \int_{t_1}^{t_2} v(t) \, dt \approx \left( \frac{1}{\Delta x \Delta z} \right) \int_{x_1}^{x_2} \int_{z_1}^{z_2} V(x, z) \, dx \, dz
\]  

(1)
The averaging operator takes the integral of the glider velocity as a function of time divides by the change in \(x\) or distance traveled during the glider flight. As seen in Equation 1, it can be re-written as velocity being an integrated function of the horizontal \((x)\) and vertical\((z)\) distance traveled divided by the overall change in horizontal and vertical distances in order to obtain the average velocity for the volume of water the glider traveled through. The modeled geostrophic data then was used to illustrate the differences between the average expected geostrophic flow and the average observed water velocities by the glider.

**Geostrophic Velocity**

The geostrophic velocity was calculated using MATLAB® to derive the relationship between the horizontal pressure gradient and Coriolis parameter on a moving region of water, which states that geostrophic current can be found by considering the volume water of speed, \(u\), in the Coriolis force equation (Equation 2) to be the geostrophic velocity, \(v\), which is balanced by the horizontal pressure gradient force (Ocean Circulation 1989) as seen in Equation 3.

\[
F_C = m f u \\
f = 2\Omega \sin \phi
\]

\[
\rho \left( \frac{Du}{Dt} \right) - fv = - \left( \frac{\partial P}{\partial x} \right) + \left( \frac{\partial \tau_x}{\partial z} \right)
\]

Equation 2 illustrates the overall force balance between geostrophic and hydrostatic forces in the dominant east-west direction denoted by the direction of windstress \((t)\) over a vertical \((z)\) water column. The direction of the pressure gradient (horizontal, \(x\)) also demonstrates east-west bias of the force balance in Equation 3.
The previous equation can be split into two specific force balances; geostrophic and hydrostatic. As seen in \textit{Equations 4 and 5a}, the geostrophic balance containing the geostrophic velocity term, $v_g$, can be re-written in order to equal $v_g$.

$$f v_g = \left( -\frac{1}{\rho} \right) \frac{\partial P}{\partial x}$$

$$v_g = \left( -\frac{1}{\rho_f} \right) \frac{\partial P}{\partial x}$$ (4)

$$\frac{\partial P}{\partial z} = -\rho g$$ (5a)

$$P(0) + P(x, z) = \int_{z_o}^{0} \rho g \, dz$$ (5b)

The hydrostatic balance (\textit{Equation 5a}) demonstrates that the changing pressure ($P$) gradient over the $x$-direction equals the density ($\rho$) of a volume of water multiplied by the gravity constant ($g$) over a vertical volume ($dz$). \textit{Equation 5b} is a re-written hydrostatic equation which states that the change in pressure ($P$) from the surface ($P(0)$) to the seafloor ($P(x, z)$) is equal to the integral of density ($\rho$) and gravity ($g$) over the area $dz$. This equation defines a term for pressure ($P$) using the water’s density and the gravity constant which can then be plugged into the $v_g$ equation (\textit{Equation 3}) to yield:

$$v_g = \left( -\frac{1}{\rho_f} \right) \frac{\partial P}{\partial x} = \left( -\frac{1}{\rho_f} \right) \frac{\partial}{\partial x} \left( \int_{z}^{0} \rho g \, dz \right)$$ (6)

\textit{Equation 6} gives the instantaneous geostrophic velocity for one particular area of water therefore as previously seen in \textit{Equation 3} the geostrophic velocity must also be averaged to adequately represent the flow over one dive period. \textit{Equation 1} is therefore re-written for an estimated geostrophic velocity, $V_g$ in \textit{Equation 7}. 

$$v_g = \left( -\frac{1}{\rho_f} \right) \frac{\partial P}{\partial x} = \left( -\frac{1}{\rho_f} \right) \frac{\partial}{\partial x} \int_{z}^{0} \rho g \, dz$$ (6)
\[ V_g = \left( \frac{1}{\Delta x \Delta z} \right) \int_{x_1}^{x_2} \int_{z_1}^{z_2} v_g \, dx \, dz = \left( \frac{1}{\Delta x \Delta z} \right) \int_{x_1}^{x_2} \int_{z_1}^{z_2} \left( -\frac{1}{\rho g f \partial x} \frac{\partial P}{\partial z} \right) \, dx \, dz \]  

The asterisk (*) on \( dz^* \) discerns between the outer, double integral and inner integral of the averaging operator once the \( v_g \) (Equation 6) term is placed into Equation 7. The inner \( v_g \) integral can be replaced with an equivalent equation displayed previously in Equation 5 eliminating the need for the inner integral over \( dz^* \) since the integral density(?) and gravity(\( g \)) over \( z \) is also equal to the partial derivative of pressure(\( P \)) over \( x \).

\[ V_g = \left( \frac{1}{\Delta x \Delta z} \right) \int_{x_1}^{x_2} \int_{z_1}^{z_2} \left( -\frac{1}{\rho f} \frac{\partial P}{\partial x} \right) \, dx \, dz \]  

\[ V_g = \left( -\frac{1}{\rho f} \right) \left( \frac{1}{\Delta x \Delta z} \right) \int_{-H}^{P(x_2, z) - P(x_1, z)} dz \]  

Equation 8a illustrates the averaging operator in its re-written form while Equation 8b demonstrates that the partial derivative of pressure over \( x \) is a direct integral to the double integral over \( x \) therefore the equation can be re-written to include only the singular \( z \) integral. This integral represents the vertical integration (\( dz \)) of pressure from one point \( (x_1) \) to another \( (x_2) \) where \( z_1 \) is considered the sea floor, or \( -H \), and \( z_2 \) represents the surface of the water or zero, \( 0 \).

In order to integrate the vertical water column traveled during this particular velocity the pressure terms (\( P \)) are separated into two different equations which are both equal to a previously described Equation 4b where pressure is also equal to the integral of density and gravity over \( dz \) (seen in Equation 9).
A density plane is fit to the equation (Equation 10) since the pressure is representative of the entire water column which after integration yields Equation 11 which is the same for each horizontal point \(x\) at which pressure is being calculated.

\[
\frac{P(x_2, z)}{P(x_1, z)} = -g(ax_1 + c)z - \frac{b_z^2}{2}
\]  

(11)

The solution for pressure at points \(x_1\) and \(x_2\) is then placed back into Equation 8b and yields Equation 12a:

\[
V_g = \left(-\frac{1}{\rho_f}\right) \left(\frac{g}{\Delta x \Delta z}\right) \int_{-H}^{0} \left[ (ax_2 + c)z - \frac{b_z^2}{2} \right] + \left[ (ax_1 + c)z + \frac{b_z^2}{2} \right] \, dz
\]  

(12a)

\[
V_g = \left(-\frac{1}{\rho_f}\right) \left(\frac{g}{\Delta x \Delta z}\right) \int_{-H}^{0} az(x_1 - x_2) \, dz
\]  

(12b)

Once the equation is simplified to 12b the final vertical integration can place in order to find averaged geostrophic velocity \(V_g\) in relation distance to the sea floor \(H\), Coriolis parameter \(f\) and the density of the water column being modeled \(\rho_f\). Upon solving the final integral the average geostrophic velocity is found to equal:
\[ V_g = -\left(\frac{g\alpha H}{\rho f^2}\right) \]  

(12c)

It can be determined that the average geostrophic flow of water is dependent upon the parcel of water's distance from the sea floor \(H\), Coriolis parameter \(f\), gravity constant \((9.81 \text{ m/s}^2)\), density \(\rho\) and finally the constant \(\alpha\) \((\text{kg/m}^4)\) which is related linearly to the density (in this case potential density) of the water column versus the distance traveled (km) at the given velocity.

![Figure 6: Illustrates the linear relationship of the constant \(\alpha\) \((-0.0131 \text{ kgm}^{-4}\)) as the slope of a linear fit to the glider travel of potential density (kg/m\(^3\)) versus distance traveled (km) for one six hour dive period (specifically for the 20202:25277 indices). The geostrophic velocities are modeled using the distance traveled and density of the water column through which the glider was traveling.](image)

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Geostrophic velocities were then calculated for each corresponding six hour dive period in which each had its own 'a' value from the linear relationship from the particular graph of density and distance for that period. *Equation 12c* was used for all averaged geostrophic velocity calculations until they could be compared to water velocities (V) determined observationally by the glider. Figure 7 compares the glider observed velocities and geostrophic velocities in the direction of the geostrophic current, in other words the vector component of measured glider velocities that is in the same direction of the geostrophic flow is being compared to the calculated geostrophic velocities.

![Figure 7: The comparison of glider observed velocities (V) and geostrophic velocities (V_g) in the direction of the geostrophic current.](image-url)
4. Discussion

The calculated geostrophic velocities helped add to the understanding of oceanographic processes over the continental shelf along the NH transect as well as provide new data concerning geostrophic currents and their presence in observed water velocities that the glider measures. Figure 7 is the resulting graph of modeled values for geostrophic currents present along the glider tracks traveled during August of 2006. The observed glider velocities were compared to the averaged geostrophic velocities in the direction of the geostrophic current along that particular glider track, therefore the portion of glider velocities compared were only the vector component which was in the same direction as the geostrophic velocity.

The geostrophic velocity data was computed using the potential density data from the glider which proved to be an easy method of incorporating glider data into the modeled geostrophic values. According to Figure 7, there were geostrophic currents present in the water velocities as observed by the glider during it's travel. They are especially prevalent during the latter portion of the gliders travel from about 45 on the x-axis onward where the geostrophic data matches observed velocities well. However there is noticeable difference in the fit between the two data sets when overlaying $V_g$ (geostrophic) and $V$ (glider) onto one another (in Figure 7). There are several possibilities one of which is the use of the glider observed density(?) data in the geostrophic calculation.

As seen in Figure 6, each group of density data that corresponded to a 6hour glider dive period used a linear regression to fit a line to the density data versus distance traveled in order to obtain the slope of the given equation of the fitted line. The resulting
slope (or 'a' value) may in fact not represent well the overall relationship between the
density and the distance. Therefore a parabolic fitted line may have been needed in order
to obtain a polynomial which was a better representation of the water density over that
period. Future work on this subject could include finding a better fit to the data as well as
a quantitative analysis which would yield a numerical value for the fit between $V_g$ and $V$.

Other possible explanations for the difference in fit could perhaps be due to bias
in glider data from variations in surface conditions such as wind stress and the activity of
the Columbia River plume. Another variable to consider is variation in the Ekman
transport occurring in the water through which the glider traveled which is not normally
constant during an entire glider transect.

5. Conclusion

The comparison of glider observed water velocities and calculated geostrophic
currents is needed in order to better understand the role of geostrophic currents in coastal
processes. Since geostrophy is the dominant oceanic process on the planet a better
understanding of it's (level of) interaction with coastal processes would allow for better
models to be created for various coastal events such as Oregon's prominent upwelling
season. Future work on this topic would include a quantitative analysis of the fit between
glider velocities and geostrophic velocities to determine how to better calculate a
geostrophic model to fit the observed velocities.
References


