Mayan Math

Adapted from: "Breaking the Mayan Code – Mayan Math"
from *Math and Science Across Cultures*

Objective:
Club members will explore the method of counting and representing numbers of the Maya. They will decode a few pages from a pre-Columbian (from before European contact) Mayan book. The investigative problem-solving skills required to decipher the Mayan code are similar skills to those required to interpret the patterns found in other foreign systems like fish populations, or satellite images. These activities all require club members to explore a way of thinking that is different from the way they are accustomed to think and that sometimes things that people take for granted need to be reconsidered.

Note: The book this activity is adapted from is an excellent resource. It has many examples of how different cultures develop different ways of mathematical and scientific thinking and how the culture you live in shapes how you view the world. The number system we use is not necessarily the best, or even the easiest, it's just what we're all used to.

Materials:
Copies of the Dresden Codex
Calculators (optional, but it makes the project easier)

Discussion:

The Maya
The pages club members will decipher are from one of the few surviving Mayan books now called the Dresden Codex. It is called the Dresden Codex because the Codex (plural codices) ended up in Dresden (a city in Germany near the border with the Czech Republic). The Maya made books (codex is a name the Europeans invented) out of pounded fig-tree bark that was treated with lime and then covered in a thin layer of plaster. They were painted with bright colors, folded up accordion-style, and bound with wood covers.
Color photos of the Dresden Codex can be found on the web – here's one site where you can find it http://www.famsi.org/mayawriting/codices/dresden.html
Most of the Maya's books were destroyed in the 1500s by the Spanish conquistadores. All that is left of the Mayan written records is either carved in stone or in the 4 surviving codices.

Ask club members what they know about the Mayan civilization (some club members may have Maya roots).

There has been a lot of speculation and predictions about the ancient Maya culture, but there is significant debate among scholars and amateurs who study ancient Mayan ruins. There are many competing theories that can be found on the web.

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To help club members see how easy it is for archaeologists and anthropologists to make mistakes based on their own cultural assumptions here are a few references. (I strongly recommend reading these two stories. I am always reminded of them when I watch a TV documentary on ancient life.)

http://www.babinlearn.com/pdf%20files/Cinderella/Nacirema.pdf
Or the book *Motel of the Mysteries* by David MacAulay

While a lot of the 'information' about the ancient Mayan society is debatable, some basic things seem fairly certain.

The ancient Mayan society was far reaching and lasted a long time. It extended through southern Mexico, the Yucatán Peninsula, Belize, Guatemala, Honduras, and El Salvador. The Maya were spread over about 120,000 square miles, roughly the size of New Mexico. The society was organized into city-states with road systems connecting the communities. The first evidence of Maya living in the area date back to approximately 2600 B.C.E. (B.C.E. stands for Before the Common Era and means the same thing as B.C. Since B.C. stands for Before Christ many non-Christians and some academics use the more neutral B.C.E.) Mayan society appeared to be at its best around 200 C.E. – 800 C.E. (C. E. stands for Common Era, is equivalent to A.D. and is used for the same reason as B.C.E.). Modern scholars refer to this era as the Classical Period.

Around 761 C.E. something happened to cause the Mayan society to collapse and many city-states were abandoned. I've listed some of the hypotheses about this collapse. Some researchers think that severe drought and crop failure could have caused the collapse, while others attribute the collapse to a period of intense warfare. These two explanations are compatible with each other. Severe drought and crop failure could induce a period of warfare. Another hypothesis is that the stucco the Maya used to coat their buildings was environmentally an unsustainable process. Stucco is a cement mixture made of sand and lime. Lime (calcium oxide - CaO) is made by burning limestone or seashells (calcium carbonate - $CaCO_3$) in very hot fires.

$$CaCO_3 + \text{heat} = CaO + CO_2$$

In order to make lime the Maya had big furnaces that they powered with wood from the surrounding forests. Archaeologists have found shell fragments in Mayan furnaces hundreds of miles from the ocean. Some researchers think that they produced so much lime that they deforested the areas around their cities causing all sorts of environmental and ecological problems. Some researchers hypothesize that by deforesting their environment the ancient Maya caused the droughts and crop failures that others have suggested. There are some other hypotheses that can be found in literature or on the web.

After the Classical Period not much is known about the Maya until the 1500s when the Spanish arrived in Central America. The Spanish saw the Mayan writing system as "lies of the devil" and systematically destroyed most of the Mayan literature. The Spanish enslaved many Maya and tried to destroy the Mayan culture.

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Ancient Mayans studied arts and sciences and knew the cycles of the earth, moon, and other planets. Evidence from their calendars indicates that the Maya believed that the Earth was millions of years old, Europeans of the time believed the Earth to be only a few thousand years old.

What was happening in other parts of the world during the Mayan Classical Period? What were cities like in Europe? Asia? Africa? South America? Did these societies have written language? Is there any evidence of their architecture left?

Numerical Bases:

Pass out copies of the pages of the Dresden Codex and let club members look at them for a while. Brainstorm as a club or in small groups ways to go about deciphering the Mayan number system. What are things you need to figure out to break the code?

What base did the Maya used in their numerical system? Different cultures use (and have used) counting systems with different numerical bases to suit their needs. We use base ten. Ask club members why they think we use base ten.

Some cultures count on only one hand and use base 5 for counting (in some African languages the word for five is the same as the word for 'hand full'). Other cultures count using their fingers and toes and use a base 20. Some Native American cultures use base eight. Ask club members why they might use base eight. Hint: count the spaces between all of your fingers. Computers use binary numbers or base 2. Why? In base 2 all numbers can be displayed as a series of 1's and 0's, or 'on' and 'off'. Early computers had switches (like light switches) that programmers flipped to create the numbers that they wanted. Before floppy discs computer programs were written in code and then transferred onto stacks of punch cards (a punch = 1, and no punch = 0) to be read by the computer. Everything in the computer world is expressed in powers of 2. Each digit is called a bit and sets of 8 bits are called bytes. A MB is a megabyte or 100,000 bytes, a GB is a gigabyte or 1,000,000 bytes. Memory sticks come in a variety of sizes: 64 MB (2^6), 128 MB (2^7), 256 MB (2^8), 512 MB (2^9), etc. Satellites have enough memory so that each pixel can have 256 possible values.

Ask club members what numerical bases would be most likely for cartoon humans who usually have 4 appendages on each limb? Likely bases would be 4 (one hand), 8 (two hands), or 16 (all 4 limbs). Another likely system would be based on 3's if they chose to count the gaps between fingers instead of the fingers themselves.

An example:

The table below has the number 459 as it would look in different base systems. Ask club members why they think a different symbol is needed to represent the value 19 in base 20? What other numbers in base-twenty would need symbols that don't exist in our base 10 counting system?

If you wrote the number "19" in base 20 it would mean nine 1's and 1 20 (that would equal 29 in base 10).

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BASE 20		
400's	20's	1's
1	2	(symbol for 19)

BASE 10		
100's	10's	1's
4	5	9

BASE 5			
125's	25's	5's	1's
3	3	1	4

BASE 2								
256's	128's	64's	32's	16's	8's	4's	2's	1's
1	1	1	0	0	1	0	1	1

Interpreting the Pages from the Dresden Codex:

First, which direction is up? What are some clues that help you decide which direction is up? There are two drawings of human figures in the document. One way to start would be to hypothesize that the human figures should be seated upright in the document.

Now you can start looking for patterns in the writing. Sometimes holding the document at arms-length and squinting can help you find patterns. Some of the glyphs (symbols used for language – like hieroglyphics) are complex and some of the symbols seem simpler. It is known that there are both numbers and words in this text, you might start by deciding which type of symbols might represent numbers and which might represent numbers. Which do you think is which? What clues did people use to make their hypotheses?
(The more complex glyphs are the symbols for words and the symbols with lines and dots are the numbers.)

Reading the Numbers:

Now that you know which symbols are the numbers, you can begin to figure out what the number symbols mean. How can you figure out what the bars and dots represent? What are some hypotheses?

The dots are small and simple; one hypothesis could be that the dots are the 1's. What is the largest number of dots together in one line? Notice that there are never more than 4 dots in a row. If there are never more than 4 dots in a row and they each equal 1, then what number would the bar logically represent? How many of them in a row are there? Are there other possible numbers the bars and dots could represent? Try out different values for the bars and see what numbers are created? Which number for the bar makes most sense? Try writing the

numbers out in your hypothesized number system. Can you make all the numbers between 1-20? (the bars represent the number 5).

What base does this system use?

Now that you can decipher the symbols used to create numbers, you need to figure out what base the Maya used. A group is a single row of dots and the bars under it. Some numbers have just dots, or just bars. The largest group consists of 4 dots and 3 bars. What number does this represent? If the bars equal 5 and the dots equal 1, then what number is 4 dots and 3 bars? Does this help you decide what base the Mayan system uses? If our previous hypothesis that bars equal 5 and dots equal 1 then the largest numeral is 19, and the system is a base-twenty system.

How would you write bigger numbers using this system? How do we write numbers bigger than 9 in our system? You have to rely on a place system. Refer to the chart to determine what the places are in the Mayan system. Each group of symbols is a digit in the Mayan system.

Which direction do numbers go in?

In order to interpret these numbers you need to know how the Maya read. Did their writing go from left to right like English, Spanish, and other European based languages? Or did their writing go right to left like Hebrew, Arabic, and other languages developed in the Middle East? Did they read from top to bottom like Japanese? Or did they read from bottom to top? Choose the hypotheses you think are best, and try them out. Compute the numbers for each possible system and see which makes the most sense. Or have different groups try different systems and compare results. Remember that this is in base-20 system, the first digit is the 1's (20^0) the second digit is the 20's (20^1) the third digit is the 400's (20^2) what would the 4^{th} (20^3) digit be?

If you read the digits horizontally, the numbers are astronomically huge! The 6^{th} digit in base-20 is the 3,200,000's place! This is probably not the way these numbers should be read. Top to bottom, or bottom to top seems more likely.

From top 1's to bottom 400's (and left to right) the numbers are:

5934	4555	3535	2156	1136	4117

From bottom 1's to top 400's (and left to right) the numbers are:

5934	6151	6328	6545	6722	6910

Is there a pattern in either of these number sets? The set where 1's are at the bottom has more of a pattern, the numbers increase fairly steadily from left to right. Do you think that there's more meaning in a set of numbers that has a pattern?

What does it mean?

Now you know that the Maya used a base-20 counting system, and that numbers are read from bottom to top, and maybe left to right. So what is this codex all about? One tool mathematicians

use when looking for meaning in groups of numbers is to find the difference between two numbers.

6151 – 5934 = 217
6328 – 6151 = 177
6545 – 6328 = 217

Figure out the rest of the differences.

217 177 217 177 188

What could this mean? Try adding the pairs of numbers together. These numbers are between 354 and 434 and one is 365. Is that a familiar number?

Each number is close to half of 365 - the number of days in a year. The number 177 is exactly 6 lunar months of 29 ½ days each. Archeologists believe that the Dresden Codex is a record of astronomical observations (or predictions) of lunar eclipses, which happen about once every six months. Maya numbered days from certain points in compatible calendar systems and the numbers were the dates. Maya had 3 different calendar systems, and days when they all coincided symbolized the end of an era, much like our millennia, each era was a new "long count".